QUESTION

Consider the points $L = (1, 1, 1)$, $M = (1, -1, 2)$ and $N = (-1, 2, 3)$.

(i) Write down the vectors $\mathbf{u} = LM$ and $\mathbf{v} = LN$ and find their lengths.

(ii) Calculate the dot product $\mathbf{u}.\mathbf{v}$ and the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

(iii) Compute the cross product $\mathbf{u} \times \mathbf{v}$ and use it to write down the vector equation of the plane Π_1 containing the three point L, M, n. What is the equation of the plane in terms of x, y, z coordinates?

(iv) Write down the vector equation of the plane Π_2 parallel to Π_1 and passing through the origin. Find the distance between the planes Π_1 and Π_2.

ANSWER

(i) $\mathbf{u} = (0, -2, 1)$, $|\mathbf{u}| = \sqrt{5}$ $\mathbf{v} = (-2, 1, 2)$, $|\mathbf{v}| = 3$

(ii) $\mathbf{u}.\mathbf{v} = 0$ so $\theta = \frac{\pi}{2}$

(iii) $\mathbf{u} \times \mathbf{v} = (-5, -2, -4)$

Π_1 has equation $(-5, -2, -4).(\mathbf{x} - (1, 1, 1)) = 0$ so $5x + 2y + 4z = 11$

(iv) Π_2 has equation $\mathbf{v}.\begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix} = 5x + 2y + 4z = 0$

Distance between the two planes $=|r(\mathbf{u} \times \mathbf{v})|$ where $(-5r, -2r, -4r)$ lies on Π_1 so $r = \frac{-11}{45}$ hence $|r(\mathbf{u} \times \mathbf{v}| = \frac{11}{\sqrt{45}}$.