Question

Four points in space have coordinates

$$A(1,1,0) \quad B(3,0,1) \quad C(1,0,2) \quad D(1,1,3)$$

Find the equations of two parallel planes, of which one contains A and B and the other contains C and D. Deduce the shortest distance between the lines AB, CD.

Answer

Parallel planes have the same normal vectors.
Let $\mathbf{a}$ be such a normal vector.

Then $\mathbf{a} \cdot \vec{AB} = 0$ and $\mathbf{a} \cdot \vec{CD} = 0$
So $\mathbf{a} \cdot (2, -1, 1) = 0$ and $\mathbf{a} \cdot (0, 1, 1) = 0$
$\Rightarrow 2a - b + c = 0$ and $b + c = 0$
Choose $c = -1 \, b = 1 \, a = 1$
So $\mathbf{a} = (1, 1, -1)$ is a normal vector.

The equation of the plane though
$AB : x + y - z = 2$
$CD : x + y - z = -1$
So the distance between the planes is $\dfrac{3}{\sqrt{3}} = \sqrt{3}$