Question

(a) Find the equation of a sphere centre $\mathbf{c}$ and radius a.

(b) Show that the equation of the tangent plane at a point $\mathbf{d}$ on the sphere is
$$\mathbf{r} \cdot \mathbf{d} - \mathbf{c} \cdot (\mathbf{r} + \mathbf{d}) + k = 0$$
where k is some scalar to be determined.

Answer

(a) $|r - c|^2 = a^2$ or $(\mathbf{r} - \mathbf{c}) \cdot (\mathbf{r} - \mathbf{a}) = a^2 \quad \mathbf{r} \cdot \mathbf{r} - 2\mathbf{r} \cdot \mathbf{c} = a^2 - c^2$

(b) a normal vector to the plane i s$\mathbf{d} - \mathbf{c}$ so the equation is $\mathbf{r} \cdot (\mathbf{d} - \mathbf{c}) = k$
Thus the equation is $\mathbf{r}(\mathbf{d} - \mathbf{c}) = \mathbf{d} \cdot (\mathbf{d} - \mathbf{c})$

$$\mathbf{r} \cdot \mathbf{d} - \mathbf{c}(\mathbf{r} + \mathbf{d}) - \mathbf{d} \cdot \mathbf{d} = 0$$