Question

If f is measurable prove that for all $c \epsilon \mathbf{R}$ $\{x | f(x) = c\}$ is measurable. Is the converse of this result true?

Answer

$\{x | f(x) = c\} = \{x | f(x) \le c\} \cap \{x | f(x) \ge c\}$

The converse is not true, for example let $\mathbf{R}_+^{\mathbf{n}}$ be the half space $x_1 > 0$. Let A be a non-measurable subset of $\mathbf{R}_+^{\mathbf{n}}$. Define $f : \mathbf{R}^{\mathbf{n}} \to \mathbf{R}^*$ by

$$f(x) = \begin{cases} 0 & \text{if} \quad x \notin \mathbf{R}_+^{\mathbf{n}} \\ +\infty & \text{if} \quad x \epsilon A \\ -\infty & \text{if} \quad x \epsilon \mathbf{R}_+^{\mathbf{n}} - A \end{cases}$$