Partial Differentiation
Limits

Question

Evaluate the given limit. If the limit does not exist, explain why.

$$\lim_{(x,y)\to(0,1)} \frac{x^2(y-1)^2}{x^2 + (y-1)^2}$$

Answer

$$\lim_{(x,y)\to(0,1)} \frac{x^2(y-1)^2}{x^2 + (y-1)^2} = 0$$

This is because

$$0 \le \left| \frac{x^2(y-1)^2}{x^2 + (y-1)^2} \right| \le x^2$$

and $x^2 \to 0$ as $(x,y) \to (0,1)$.