Partial Differentiation
Limits

Question
Evaluate the given limit. If the limit does not exist, explain why.

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x^2+y^2}$$

Answer
Let $f(x,y) = \dfrac{\sin(xy)}{x^2+y^2}$.

$$\begin{aligned} \Rightarrow f(0,y) &= 0/x^2 = 0 \to 0 \\ \text{as } x &\to 0 \\ \text{But } f(x,x) &= \frac{\sin x^2}{2x^2} \to \frac{1}{2} \\ \text{as } x &\to 0 \end{aligned}$$

$\Rightarrow \lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(xy)}{x^2+y^2}$
Does not exist