

Partial Differentiation
Limits

Question

Evaluate the given limit. If the limit does not exist, explain why.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^4}$$

Answer

$$\begin{aligned} x^2 &\leq x^2 + y^4 \\ \Rightarrow \frac{x^2 y^2}{x^2 + y^4} &\leq y^2 \rightarrow 0 \\ &\text{as } y \rightarrow 0 \end{aligned}$$

$$\Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^4} = 0$$