

Partial Differentiation

Limits

Question

Evaluate the given limit. If the limit does not exist, explain why.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y^2}{2x^4 + y^4}$$

Answer

$$\begin{aligned} & \text{If } x = 0 \text{ and } y \neq 0 \\ \Rightarrow & \frac{x^2y^2}{2x^4 + y^4} = 0 \\ & \text{If } x = y \neq 0 \\ \Rightarrow & \frac{x^2y^2}{2x^4 + y^4} = \frac{x^4}{2x^4 + x^4} = \frac{1}{3} \\ \Rightarrow & \lim_{(x,y) \rightarrow (0,0)} \frac{x^2y^2}{2x^4 + y^4} \text{ Does not exist.} \end{aligned}$$