Partial Differentiation
Limits

Question
Let function f be given by

$$f(x,y) = \frac{\sin x \sin^3 y}{1 - \cos(x^2 + y^2)}.$$

Can f be defined at $(0,0)$ so that it becomes continuous there? If it is possible, explain how.
Answer

$$f(x,y) = \frac{\sin x \sin^3 y}{1 - \cos(x^2 + y^2)}.$$

Cannot be defined at $(0,0)$ to be continuous there.
This is because $f(x,y)$ has no limit as $(x,y) \to (0,0)$.
Observe that $f(x,0) = 0$, so the limit, if it did exist would have to be 0.
But

$$f(x,x) = \frac{\sin^4 x}{1 - \cos(2x^2)} = \frac{\sin^4}{2\sin^2(x^2)}$$

and $f(x,x) \to \frac{1}{2}$ as $x \to 0$.
So f cannot be defined to be continuous at $(0,0)$.