

Maths 3018/6111 - Numerical Methods

Worksheet 2

Theory

1. Perform the LU decomposition of

$$A = \begin{pmatrix} 1 & 3 \\ 4 & 16 \end{pmatrix}.$$

Use both standard factorisation methods.

2. [Additional] Write out the Thomas algorithm for a tridiagonal system.
3. Write down the general framework for iterative methods for linear systems. Give the convergence matrix. If the linear system uses the matrix A above, will an iterative method converge? [Hint: remember what to do with the diagonal entries]
4. Check which of the matrices on this sheet are diagonally dominant.
5. Briefly explain what is meant by quadrature methods based on polynomial interpolation.
6. [3018 only] Write down the contraction mapping theorem. Check that $g(x) = \cos(x)$ is contracting on the unit interval.

Coding

1. Write a code to do LU decomposition. Check the decomposition of

$$B = \begin{pmatrix} 64 & 8 & 48 \\ 24 & 28 & 53 \\ 32 & 49 & 91 \end{pmatrix}.$$

2. [Additional] Implement the Thomas algorithm for a tridiagonal system.
3. Implement the Jacobi method for linear systems. Investigate the behaviour of the method for the problem

$$B\mathbf{x} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

starting from the trivial initial guess. Should this be expected? [Hint: Check the convergence theorem] Try instead the matrix

$$C = \begin{pmatrix} \frac{119}{108} & -\frac{14}{27} & -\frac{8}{9} \\ \frac{7}{54} & \frac{46}{27} & \frac{7}{23} \\ \frac{5}{108} & \frac{1}{27} & \frac{23}{18} \end{pmatrix}.$$

4. Implement the Gauss-Seidel method, applying it to the convergent problem above. Compare the convergence rate to Jacobi.
5. [3018 only] Implement the chord method to find the root of

$$f(x) = \tan x - e^{-x}, \quad x \in [0, 1].$$