

Maths 3018/6111 - Numerical Methods

Worksheet 6

Theory

1. Explain the shooting method for BVPs.
2. Give a *complete* algorithm for solving the BVP

$$y'' - 3y' + 2y = 0, \quad y(0) = 0, \quad y(1) = 1$$

using the finite difference method. Include the description of the grid, the grid spacing, the treatment of the boundary conditions, the finite difference operators and a description of the linear system to be solved. You do not need to say which method would be used to solve the linear system, but should mention any special properties of the system that might make it easier to solve.

3. Explain how your algorithm would have to be modified to solve the BVP where the boundary condition at $x = 1$ becomes the Neumann condition

$$y'(1) = 1 + \frac{e}{e-1}.$$

Coding

1. Write a simple shooting method to solve the BVP

$$y'' - 3y' + 2y = 0, \quad y(0) = 0, \quad y(1) = 1.$$

Use `ode45` to solve the ODE, rewritten in first order form, and either a simple bisection method or the in-built Matlab `fsolve` routine to find the root. Compare your estimate against the answer

$$y(x) = \frac{e^{2x-1} - e^{x-1}}{e-1}.$$

2. Implement your finite difference algorithm above to solve this BVP, using Matlab's in-built linear system solver. Show that your result converges to the correct answer.
3. Modify your algorithm for the Neumann boundary condition above. Check that it converges to the same answer as for the Dirichlet case.