Question

The cdf of a continuous r.v. \boldsymbol{Y} is given by

$$F(y) = \begin{cases} 0, & \text{for } y < 0, \\ 2\frac{y^2}{\theta}, & \text{for } 0 \le y \le \frac{\theta}{2}, \\ 1 - \frac{2(\theta - y)^2}{\theta^2}, & \text{for } \frac{\theta}{2} \le y \le \theta, \\ 1, & \text{for } y > \theta, \end{cases}$$

where θ is a positive constant. Find the pdf of Y, and sketch the cdf and the pdf of Y.

Answer

From the relationship between pdf an cdf

$$f(y) = \frac{dF(y)}{dy} = \begin{cases} 0, & y < 0\\ \frac{4y}{\theta^2}, & 0 \le y \le \frac{\theta}{2}\\ \frac{4(\theta - y)}{\theta^2}, & \frac{\theta}{2} \le y \le \theta\\ 0, & \theta < y \end{cases}$$