**Question**

The lines $\frac{x}{a} - \frac{y}{b} = 0$ $\frac{x}{a} + \frac{y}{b} = 0$ are the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the asymptotes are at right angles the hyperbola is called a rectangular hyperbola. Find a condition on $a, b$ for this to be so. Find the eccentricity of a rectangular. Find the equation of a rectangular hyperbola referred to its asmptotes as axes.

**Answer**

$\frac{x}{a} - \frac{y}{b} = 0$ - slope $\frac{b}{a}$

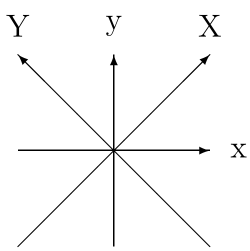
$\frac{x}{a} + \frac{y}{b} = 0$ - slope $-\frac{b}{a}$

So they are orthogonal iff $\frac{b^2}{a^2} = 1$ i.e.$b = \pm a$

So the two lines become $x = y$ $x = -y$ and the hyperbola becomes $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$

So $a^2(1 - e^2) = a^2$ $1 - e^2 = -1$ $e = \sqrt{2}$

The X-Y axis are obtained by rotation of $45°$

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\begin{aligned}
x &= X\frac{1}{\sqrt{2}} - Y\frac{1}{\sqrt{2}} \\
y &= X\frac{1}{\sqrt{2}} + Y\frac{1}{\sqrt{2}} \\
\frac{x^2}{a^2} - \frac{y^2}{a^2} &= \frac{1}{2a^2}\Big((X - Y)^2 - (Y + X)^2\Big) = 1 \\
\text{So } XY &= -\frac{a^2}{2}
\end{aligned}
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